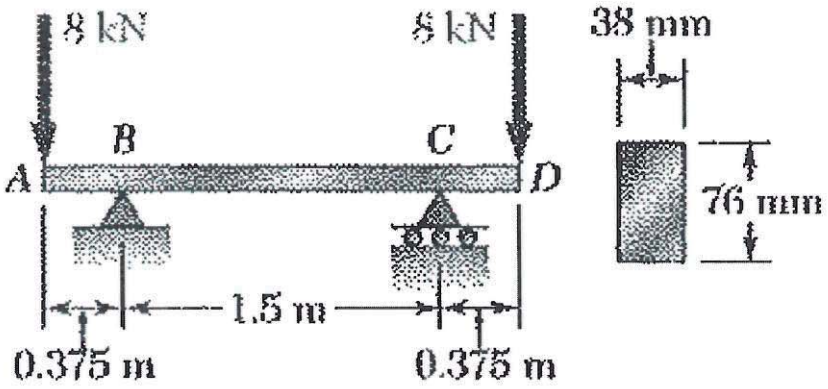
RMAII EXAMEN FINAL Q2

RESISTÈNCIA DE MATERIALS II

Examen Final Q-2 2013-14 13/06/2014

PROBLEMA 1 (3 punts)

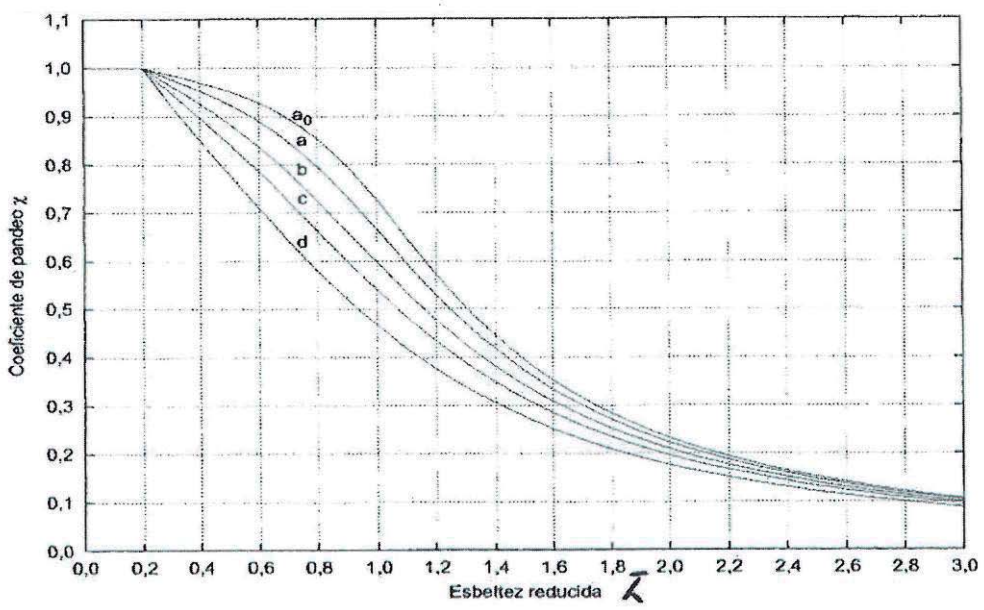
Determinar l'energia de deformació degut a les càrregues mostrades i la flexió a la barra d'acer (**E=200 GPa**).



PROBLEMA 2 (3 punts)

Un pilar de 5 m de longitud, articulat pels extrems i de secció tubular quadrada, està sotmès a una càrrega de compressió centrada. Calcular la màxima càrrega que podrà suportar i la tensió corresponent mitjançant (a) el vinclament teòric d'Euler i (b) la normativa espanyola DB-SE-A. Disposeu de la corba de vinclament que és la **a**.

Dades: f_y=300 N/mm^2; E=2,1 ·10^5 N/mm^2; γ_M=1,1
De la secció: A= 40 cm^2; I_y=I_z=450 cm^4; i_y=i_z=3,35cm



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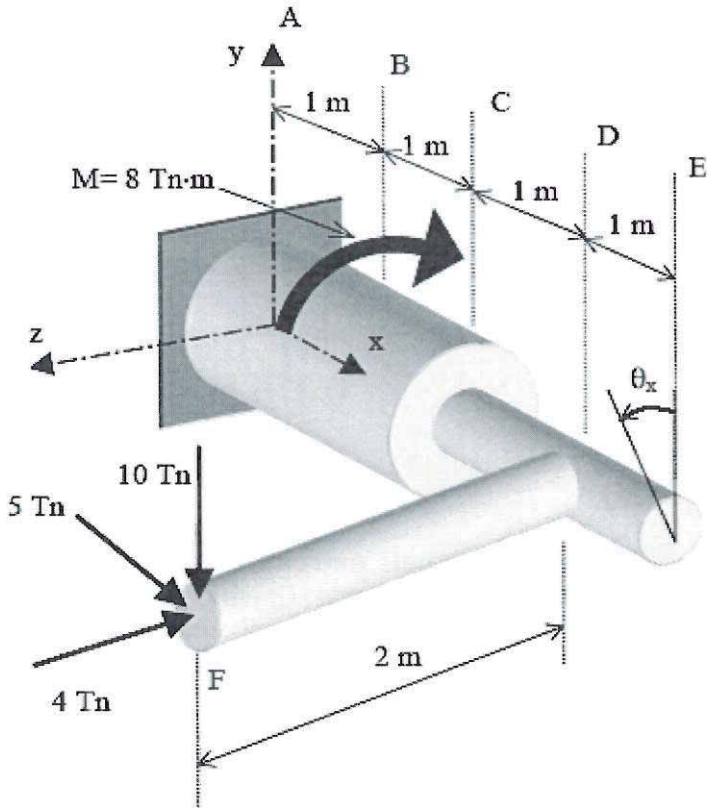
PROBLEMA 3 (4 punts)

L'estructura de la figura **encastada pel suport A** està formada per tres **rodons d'acer** soldats, les dades dels quals es troben a la figura. El moment torsor de 8Tn·m està aplicat a la secció B.



Tramo AC: ∅ = 40 cm
Tramo CE: ∅ = 10 cm
Tramo DF: ∅ = 10 cm
Material: acero
$G = 8{,}4 \cdot 10^5$ kgf/cm^2

Determinar:

1- La tensions màximes a l'encastament A (la tensió normal màxima de la secció i la tensió tallant al mateix punt).

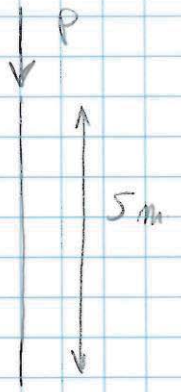
2- Dibuixar la secció A i situar aproximadament el punt calculat a l'apartat 1., i dibuixar aproximadament per on passa l'eix neutre.

3- Calcular les tensions principals a partir de les màximes de la secció A.

4- L'angle de gir a la secció E (l'estructura gira al voltant de l'eix x)

Es recomana realitzar l'exercici en unitats de kg i cm.

2



$I = 450\ cm^4 \cdot \frac{10000\ mm^4}{1\ cm^4}$

$A = 40\ cm^2 \cdot \frac{100\ mm^2}{1\ cm^2}$

a) Utilitzant teoria d'Euler:

$P_{crítica} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 \cdot 450 \cdot 0000}{5000^2} = \boxed{373071\ N}$

b) Mètode expeditiu:

$\sigma = \frac{P}{A} = \frac{373071}{4000} = \boxed{93{,}267\ MPa}$

$P = \frac{\pi^2 E I}{L^2} = 373071\ N$

$\lambda = \sqrt{\frac{A \cdot f_y}{P}} = \sqrt{\frac{4000 \cdot 300}{373071}} = 1{,}79 \rightarrow$ taula a $\rightarrow \chi = 0{,}27$

$P = \chi A \frac{f_y}{\gamma_m} = 0{,}28 \cdot 4000 \cdot \frac{300}{1{,}1} = \boxed{305454{,}54\ N}$

$\sigma = \frac{F}{A} = \frac{305454{,}54}{4000} = \boxed{76{,}36\ MPa}$

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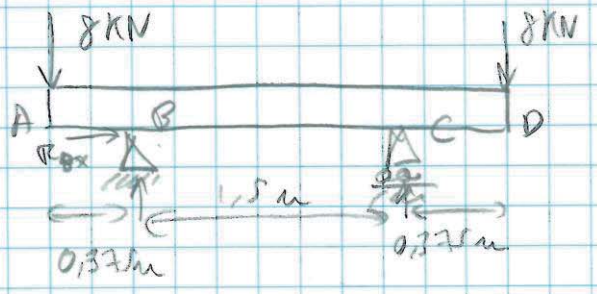


$E = 200GPa = 200 \cdot 10^9 \ N/m^2$

$\sum F_x = 0 \rightarrow R_{Bx} = 0$

$\sum F_y = 0 \rightarrow R_{By} + R_{Cy} = 16.000$

$\sum M_B = 0 \qquad R_{Cy} \cdot 1,5 + 8000 \cdot 0,375 = 8000 \cdot 1,875$

$R_{Cy} = 8000 N$

$R_{By} = 8000 N$

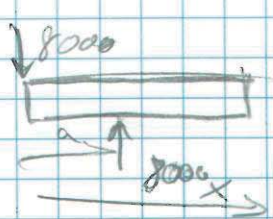
Tramo AB = Tramo CD

$M = 8000 \cdot x$

$$U_{AB} = \int_0^x \frac{(8000 \cdot x)^2}{2EI} dx = \frac{8000^2}{2EI} \int_0^x x^2 dx = \frac{8000^2}{2EI} \left[\frac{x^3}{3}\right]_0^x = \frac{8000^2 x^3}{2EI\ 3} = \frac{8000^2 x^3}{6EI} =$$

$$= \frac{8000^2 \cdot 0,375^3}{6 \cdot 200 \cdot 10^9 \cdot \frac{0,03 \cdot 0,07^3}{12}} = 2,023 \ J$$

Tramo BC



$M = 8000x - 8000(x-a) = 8000x - 8000x + 8000a$

$$U_{BC} = \int_0^a \frac{M^2\,dx}{2EI} = \frac{(8000a)^2}{2EI}\int_0^{1,5} dx = \frac{8000^2 a^2 a}{2EI} =$$

$$= \frac{8000^2 a^3 x}{2EI} = \frac{8000^2 \cdot 0{,}375^2 \cdot 1{,}5}{2 \cdot 200\cdot 10^9 \cdot \frac{0{,}038 \cdot 0{,}078^3}{12}} = 24{,}274$$

$$U_t = U_{AB} + U_{BC} + U_{CD} = 2{,}073 + 24{,}274 + 2{,}073 = \boxed{28{,}32\ \text{J}}$$

(3)

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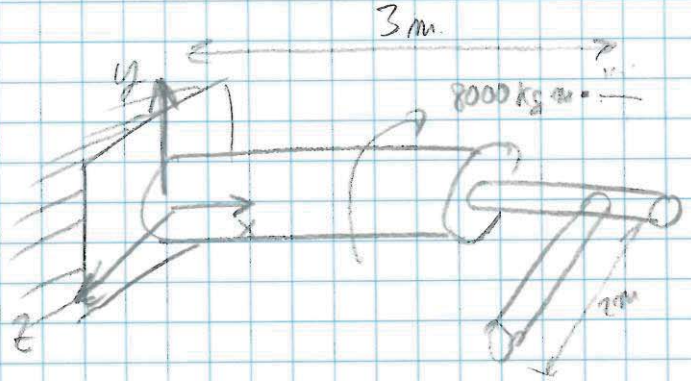


1,1 T

$\Sigma F_y \rightarrow -F_1 \rightarrow \tau_{tall} = \tau = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{98100}{\frac{\pi}{4}(400)^2} = 1,041\ MPa$ ✓

$\Sigma F_z \rightarrow -F_3 \rightarrow \tau_{tall} \rightarrow \tau = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{39240}{\frac{\pi}{4}(400)^2} = 0,416\ MPa$

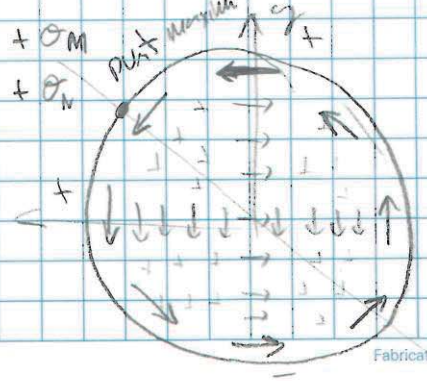
$\Sigma F_x \rightarrow +F_2 \rightarrow \sigma_{tracc} \rightarrow \sigma_N = \frac{49050}{\frac{\pi}{4}(400)^2} = 0,39\ MPa$ ✓

$\Sigma M_x \rightarrow -1,948\cdot10^7\ N\,mm + (98100\ N \cdot 2000\ mm) = 1,176\cdot10^8\ N\,mm$ OK.

$\tau_0 = \frac{1,176\cdot10^8}{\frac{\pi}{32}(400)^4} \cdot 200 = 9,36\ MPa$ 10 !

$\Sigma M_y \rightarrow (49050\ N \cdot 2000\ mm) + (39240 \cdot 3000\ mm) = 2,158\cdot10^8\ N\,mm$ ✓

$\Sigma M_z \rightarrow -98100 \cdot 3000 = 2,943\cdot10^8\ N\,mm$ ✓



$\sigma_M = \frac{\sqrt{M_y^2 + M_z^2}}{\frac{\pi}{64}D^4} \cdot \frac{D}{2} = \frac{\sqrt{(2,158\cdot10^8)^2 + (2,943\cdot10^8)^2}}{\frac{\pi}{64}\cdot(400)^4}$

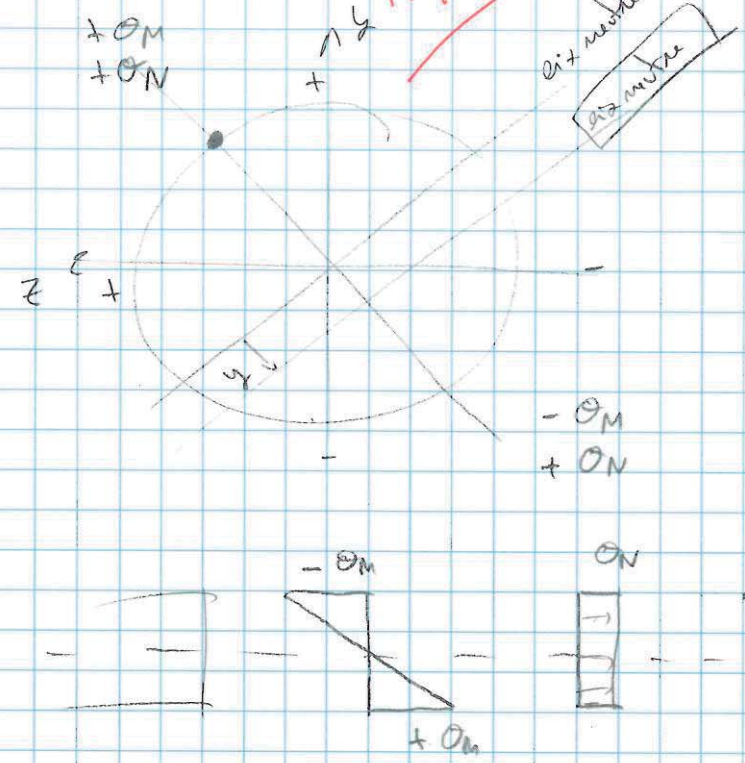
$= 58,08\ MPa$ ✓

1 $\sigma_{max} = \sigma_M + \sigma_N = 58{,}08 + 0{,}39 = 58{,}47$ MPa

$\tau_0 = 9{,}31$ MPa

2,5

2,5

2 

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$$\sigma_{1,2} = \frac{58{,}47}{2} \pm \sqrt{\left(\frac{58{,}47}{2}\right)^2 + 9{,}31^2} = 29{,}235 \pm 30{,}68$$

$\sigma_1 = 59{,}915$ MPa

$\sigma_2 = -1{,}445$ MPa

$\tau = 30{,}68$ MPa

1,25

$\phi_T = \phi_1 + \phi_2$

4 $$\phi_1 = \frac{T \cdot L}{J G} = \frac{1{,}17 \cdot 10^3 \cdot 3000}{\frac{\pi}{32} 100^4 \cdot 82404} = 1{,}169 \cdot 10^{-3}$$

$$G = 8{,}4 \cdot 10^5 \ kg/cm^2 \cdot \frac{9{,}81 N}{1 kg} \cdot \frac{1 cm^2}{100 mm^2} = 82404 \text{ MPa}$$

$\phi_T = 0{,}29$ rad

$$\phi_2 = \frac{T \cdot L}{J G} = \frac{1{,}17 \cdot 10^3 \cdot 20000}{\frac{\pi}{32} 100^4 \cdot 82404} = 0{,}289$$

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Full núm. | Nombre total de fulls | Qualificació 6 (0-6)

Falta el Prob. 3 //

1r. Cognom | 2n. Cognom | Nom | D.N.I.

Especialitat: | Curs: Q6 Grup: M65

Assignatura: | Data: 13 / 06 / 14

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Problema 1

$$I = \frac{b \cdot h^3}{12} = \frac{0'038 \cdot 0'076^3}{12} = 1'39 \cdot 10^{-6}\ m^4$$

$$U_{a,b} = \int_0^{0'375\,d} \frac{F^2 \cdot x^2 \cdot dx}{2 \cdot E \cdot I} = \frac{F^2}{2 \cdot E \cdot I} \int_0^d x^2 \cdot dx = \frac{F^2}{6 \cdot E I} \cdot \left[x^3 \right]_0^d = \frac{F^2 \cdot d^3}{6 \cdot E \cdot I}$$

$$U_{a,b} = U_{c,d} = \frac{8000^2 \cdot 0'375^3}{6 \cdot 200 \cdot 10^9 \cdot 1'39 \cdot 10^{-6}} = 2'0232\ J$$ ✓

$$U_{b,c} = 2 \int_0^{0'175} \frac{M^2\, dx}{2EI} = \frac{2 \cdot 3000^2}{2 \cdot 200 \cdot 10^9 \cdot 1'39 \cdot 10^{-6}} \cdot \left[x \right]_0^{0'175} =$$

$$M = R_A \cdot x - R_B \cdot x = 8000(0'375 + x) - 8000x = 3000$$

$$U_{b,c} = \frac{3000^2 \cdot 0'175}{200 \cdot 10^9 \cdot 1'39 \cdot 10^{-6}} = 24'2806\ J$$

$$U_{TOTAL} = U_{a,b} + U_{b,c} + U_{c,d} = 2U_{a,b} + U_{b,c} = 2 \cdot (2'0232) + 24'2806 =$$

$$U_{TOTAL} = \boxed{28'327\ J}$$

3

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Problema 2

a)

$N^* < N_{crítico}$

$N_{crítico} = \frac{\pi^2 \cdot E \cdot I_{min}}{l_k^2} = \frac{\pi^2 \cdot 2'1 \cdot 10^5 \cdot 4'5 \cdot 10^{-6}}{5^2} = 373071'0464\ N$

$\sigma_{max} = \frac{N^*}{A} = \frac{373071'0464}{40000} = 93'2678\ MPa$

$l_k = l \cdot \beta = 5 \cdot 1 = 5\ m$ $\quad N^* \leq 373071'0464$ ✓

$\sigma_{max} = 93'2678\ MPa$ ✓

$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{crit}}} = \sqrt{\frac{40 \cdot 300 \cdot 10^6 \cdot 10^{-4}}{373071'464}} = 1'793 \rightarrow \chi \simeq 0'275$

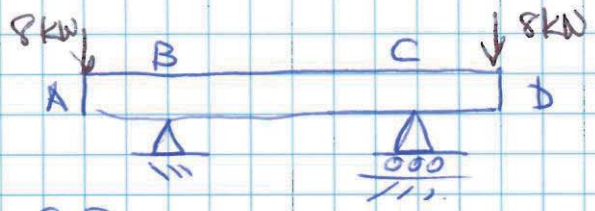
$N_{b,cd} = \chi \cdot A \cdot f_y / \gamma = 0'275 \cdot 40 \cdot 10^{-4} \cdot 300 \cdot 10^6 / 1'1 = 300000\ N$

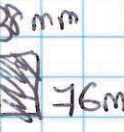
$N^* \leq N_{b,cd} = 300000\ N$

$\sigma_{max} = \frac{N^*}{A} = \frac{300000}{4000} = 75\ MPa$

3

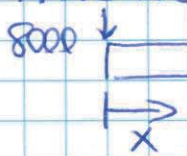
PROBLEMA 1. $E = 200\,GPa$





$$J = \frac{1}{12} \cdot b \cdot h^3 = \frac{1}{12} \cdot 38 \cdot 76^3 = \frac{1}{12}\, 0{,}038 \cdot 0{,}076^3 = 1{,}39 \cdot 10^{-6}\ m^4$$

TRAMO AB = TRAMO C-D:



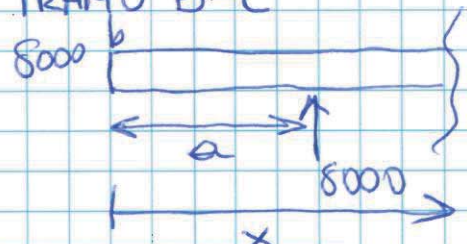
$M = 8000 \cdot x$

$$U = \int_A^B \frac{8000^2 \cdot x^2}{2 \cdot 200 \cdot 10^9 \cdot 1{,}39 \cdot 10^{-6}}\, dx = \int_A^B 115{,}11\, x^2\, dx =$$

$$= \left[\frac{115{,}11\, x^3}{3} \right]_0^{0{,}375} = 2{,}0234 \times 2 = 4{,}0468$$

TRAMO C-D

TRAMO B-C



$M = 8000\,x - 8000\,(x - a) =$

$= 8000\,x - 8000\,x + 8000 \cdot a = 3000$ (con $a = 0{,}375$)

$$U = \int_B^C \frac{M^2}{2E \cdot I}\, dx = \int_B^C \frac{3000^2}{2 \cdot 200 \cdot 10^9 \cdot 1{,}39 \cdot 10^{-6}}\, dx =$$

$$16{,}187 \int_B^C dx = 16{,}187\, x \Big]_{x=1{,}5\,m} = 24{,}2805\ N \cdot m$$

$U_T = 4{,}0468 + 24{,}2805 = 28{,}32737$

3

Resultado

PROBLEMA 2.

Pilar 5m longitud — Secció: tubular quadrada — Càrrega de compressió centrada

Calcular: màxima càrrega? — Corba vinclament a

a) tensió mitjançant vinclament teòric Euler

b) DB-SE-A

Dades: $f_y = 300 N/mm^2$ (4000 m m²) $E = 2{,}1 \cdot 10^5 N/mm^2$ $\gamma_M = 1{,}1$

Secció: $A = 40 cm^2$ $I_y = I_z = 450 cm^4$; $i_y = i_z = 3{,}35 cm$

a) Ir. Sabem que el pilar és articulat pels extrems: $\beta = 1$

$N^* \leq \chi \cdot A f_{yd}$ / b) f_y/γ_M — $N^* < N_{cr}$ Euler

$\rightarrow F \cdot \gamma \leq \chi \cdot 4000 mm^2 \cdot \frac{300 N/mm^2}{1{,}1}$

$F \cdot \gamma \leq \chi \cdot 1090{,}90 \cdot 10^3$

$N_{ey/z} = \frac{\pi^2 \cdot E \cdot I_{y/z}}{L_k^2} = \frac{\pi^2 \cdot 2{,}1 \cdot 10^5 N/mm^2 \cdot 4{,}5 \cdot 10^6 mm^4}{(\beta \cdot L)^2} = \frac{9{,}33 \cdot 10^{12} Nmm^2}{(5000)^2 mm^2}$

$\beta \cdot L$: 1

$= 373{,}2 \cdot 10^3 N$

$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{ey/z}}} = \sqrt{\frac{4000 mm^2 \cdot 300 N/mm^2}{373{,}2 \cdot 10^3 N}} = 1{,}179316$

↳ corba a de vinclament

$\chi \simeq 0{,}27$

↳ 0'275] b)

$\sigma_{cr} = \frac{N_e}{A} = \frac{373{,}2 \cdot 10^3 N}{4000 mm^2} = 93{,}3 MPa$

a) $\sigma_{cr} = 93{,}3 MPa$ tensió mitjançant Euler

$N_{ey/z} = 373{,}2 \cdot 10^3 N$ mitjançant Euler

b) A partir de la normativa hem tret que $\bar{\lambda} = 0,275 \rightarrow \bar{\chi} = 1,7932$

$N^* \leq 0,275 \cdot 4000 \cdot \frac{300}{1,1} \leq 300 \cdot 10^3 \, N < N_{cr} = 373,3 \cdot 10^3 \, N$

$$\left[\sigma_{cr} = \frac{\pi^2 \cdot E}{\left(\frac{\beta \cdot L}{i_\xi}\right)^2} = \frac{\pi^2 \cdot 2,1 \cdot 10^5 \, N/mm^2}{\left(\frac{1 \cdot 5000 \, mm}{33,5 \, mm}\right)^2} = 93,04 \, MPa \right]$$

λ_ξ

a) Se puede hacer así o $\sigma_{cr} = \frac{N_e}{A}$

$$\sigma^* = \frac{N^*}{A} = \frac{300 \cdot 10^3 \, N}{4000 \, mm^2} = 75 \, MPa$$

$\sigma^* < \sigma_{cr}$ ← $(93,3 < 75) \, MPa$

Resultats: $N^* = 300 \cdot 10^3 \, N$

$\sigma^* = 75 \, MPa$

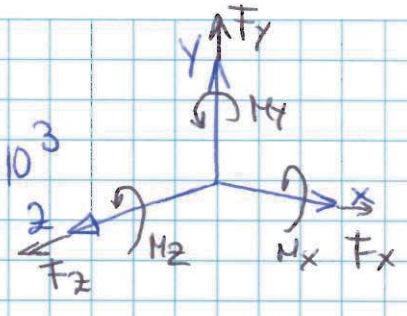
(3)

Comparacions: $N^* = 300 \cdot 10^3 \, N < N_{cr\,euler} = 373,3 \cdot 10^3 \, N$

Resultats $\sigma^* = 75 \, MPa < \sigma_{cr\,euler} = 93,3 \, MPa$

PROBLEMA 3. $\Sigma F_A = 0$;

Moment tallar $T_T \Rightarrow M = 8\,Tn.m$



$\Sigma F_x = 0$; $F_x + F_2 = 0 \rightarrow F_x = -F_2 = -5 \cdot 10^3$ kg

$M = 8000$ kg.m

$F_1 = 4000$ kg

$F_2 = 5000$ kg

$F_3 = 10000$ kg

$\hookrightarrow F_x = -5 \cdot 10^3 \rightarrow$ Força normal (tracció) σ_N

$\Sigma F_y = 0$; $F_y - F_3 \rightarrow F_y = F_3 = 10000$ kg

$\hookrightarrow$ Força tallant T_{Vy}

$\Sigma F_z = 0$; $F_z - F_1 = 0$; $F_z = F_1 = 4000$ kg $\hookrightarrow$ Força tallant T_{Vz}

~~$\Sigma M_x = 0$~~

$$\sigma_N = \frac{F_x}{A} = \frac{5 \cdot 10^3}{\frac{\pi}{4}\phi_{AC}^2} = \frac{5 \cdot 10^3 \text{ kg}}{\frac{\pi}{4} \cdot 40^2 \text{ cm}^2} = 3{,}978 \text{ kg/cm}^2$$

$$T_{Vy} = \frac{4}{3}\frac{F_y}{A} = \frac{4}{3}\frac{10000}{\frac{\pi}{4} \cdot 40^2} = 10{,}61 \text{ kg/cm}^2$$

$$T_{Vz} = \frac{4}{3}\frac{F_z}{A} = \frac{4}{3} \cdot \frac{4000}{\frac{\pi}{4} 40^2} = 4{,}244 \text{ kg/cm}^2$$

$\Sigma M_A = 0$;

$\Sigma M_x = 0$;

$\Sigma M_y = 0$; M_y

$$M_1 = F_1 \times d_A = (0, 0, -4000) \times (-300, 0, 200) = \begin{vmatrix} i & j & k \\ 0 & 0 & -4000 \\ 300 & 0 & -200 \end{vmatrix} = \begin{vmatrix} 0 \\ -1200000 \\ 0 \end{vmatrix}$$

$M_2 = F_2 \times d_A = (5000, 0, 0) \times (+300, 0, +200) = \begin{vmatrix} i & j & k \\ 5000 & 0 & 0 \\ 300 & 0 & 200 \end{vmatrix} = \begin{Bmatrix} 0 \\ +1000000 \\ 0 \end{Bmatrix}$

$M_3 = F_3 \times d_A = (0, 0, -10.000) \times (300, 0, 200) = \begin{vmatrix} i & j & k \\ 0 & 0 & -10000 \\ 300 & 0 & 200 \end{vmatrix} = \begin{Bmatrix} 0 \\ -3000000 \\ 0 \end{Bmatrix}$

$\begin{vmatrix} i & j & k \\ 0 & -1000 & 0 \\ +300 & 0 & +200 \end{vmatrix} = \begin{Bmatrix} +2000000 \\ 0 \\ +3000000 \end{Bmatrix}$

$\Sigma M_x = 0$; ~~$M + (-M_3) = 0$~~; $M_x = M - M_{3x} = 8000 \cdot 10^3$ ~~+~~

$8000\ kg \cdot m \cdot \frac{100\ cm}{1\ m} = 800 \cdot 10^3$

$+ 2000000 = 2{,}8 \cdot 10^6\ kg \cdot cm$

Torsió (π_T) 1210r

M_z

$\Sigma M_y = 0$; $M_y = 1000000 + 1200000 = 2{,}2 \cdot 10^6\ kg\,cm$ } Flexió σ_M

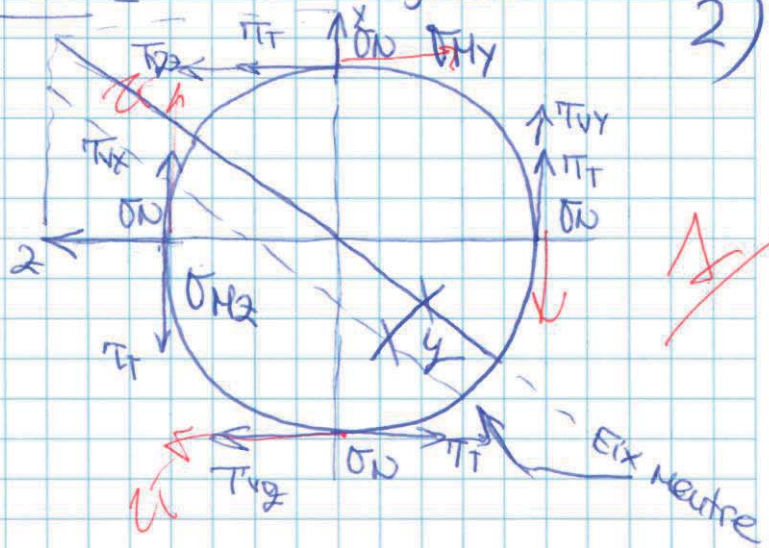
$\Sigma M_z = 0$; $M_z = 3 \cdot 10^6\ kg\,cm$ ✓ 0 (1)

$\pi_T = \frac{T}{J_0} \cdot r = \frac{10000\ kg \cdot 20\ cm}{\frac{\pi}{32} D^4_{40}\ cm^4} = 0{,}795\ kg/cm^2$

$\sigma_{My} = \frac{M_y}{I} \cdot r = \frac{2{,}2 \cdot 10^6\ kg\,cm \cdot 20\ cm}{I = \frac{\pi}{64} D^4\ cm^4} = 350{,}14\ kg/cm^2$

$\sigma_{Mz} = \frac{M_z \cdot r}{I} = \frac{3 \cdot 10^6 \cdot 20}{\frac{\pi}{64} D^4} = 477{,}46\ kg/cm^2$

2)



$\sigma_M = \frac{\sqrt{M_x^2 + M_z^2}}{\frac{\pi}{64} \cdot 40^4} \cdot 20 =$

$= 592{,}09\ kg/cm^2$ ✓

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$\sigma_m = \sigma_N + \sigma_M = 3{,}978\ kg/cm^2 + 592{,}09\ kg/cm^2 =$

$= 596{,}068\ kg/cm^2$] 1)

* $\sigma_m \rightarrow \sigma_{1,2} = \frac{\sigma_m}{2} \pm \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_T^2}$

$\sigma_{1,2} = \frac{596{,}068}{2} \pm \sqrt{\left(\frac{596{,}068}{2}\right)^2 + 0{,}795^2} =$

$= 298{,}034 \pm 298{,}035 \rightarrow \sigma_1 = 596{,}069$

$\rightarrow \sigma_2 \cong 0$

Resultat

[$\tau_{max\ secció} = \tau_T + \tau_{Vx} = 0{,}795 + 10{,}61 = 11{,}405\ kg/cm^2$] 1)

3) Tensions principals $\sigma_m \rightarrow \sigma_{1,2}$ (A sobre) *

4) ϕ ? gira al voltant de l'eix x

$\phi = \frac{T_3 \cdot L}{J \cdot G} = \frac{10000 \cdot 300}{\frac{\pi}{32} 40^4 \cdot 8{,}4 \cdot 10^5} = 1{,}42 \cdot 10^{-5}\ rad$

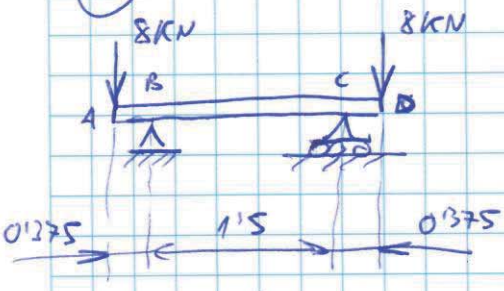
4,2

1r. Cognom | 2n. Cognom | Nom | D.N.I.

Especialitat: | Curs: | Grup: 1651

Assignatura: REGLA II | Data: 13 / 6 / 14

1



38 mm

76 mm

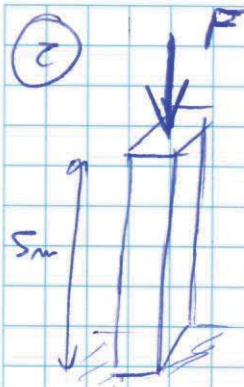
$E = 200\ GPa$

$$u = \sum u_F = u_{8kN} \cdot 2 = 7'19 \cdot 2 = \boxed{14'4}$$

$$u_1 = \frac{F^2 L}{2AE} = \frac{(8 \cdot 10^3)^2 \cdot 0'375}{2 \cdot (0'038 \cdot 0'076)^2 \cdot 200 \cdot 10^9} = \boxed{7'19\ J}$$

0'5

(2)



a) F_{max}? σ_{max}?

b) $P_{\bar{c}}$

$f_y = 300\ N/mm^2$

$E = 211 \cdot 10^3\ N/mm^2$

$\gamma_m = 1'1$

$A = 40\ cm^2 = 4000\ mm^2$

$I_z = I_y = 450\ cm^4 = 450 \cdot 10^4\ mm^4$

$i_z = i_z = 3'35\ cm$

$\sigma = \frac{F}{A}$

$$P = \frac{\pi^2 \cdot E \cdot I}{(\beta \cdot L)^2} = \frac{\pi^2 \cdot 211 \cdot 10^5 \cdot 450 \cdot 10^4}{(1 \cdot 5 \cdot 10^3)^2} = 118'75\ \frac{KN \cdot mm}{Nm}$$

$$\lambda = \frac{\beta \cdot L}{i_y} = \frac{500}{3'35} = 149'25$$

a) $\sigma_{crit_E} = \frac{\pi^2 E}{\left(\frac{\beta \cdot L}{i_y}\right)^2} = \frac{\pi^2 211 \cdot 10^{11}}{\left(\frac{1 \cdot 5}{0'0335}\right)^2} = 1'389 \cdot 10^{10} = 93'04\ MPa$

$93'04\ MPa = \frac{F}{A}$ $\qquad F_{max} = 93'04 \cdot 40 \cdot 10^2 = 372'16\ KN$

$\sigma_c = 93'04 \rightarrow \sigma_{adm} = 139'56\ MPa = 62'03\ MPa$

$\div 1'5$

$F_{adm} = 248'1\ KN$

a)

$$\lambda = \sqrt{\frac{A\,\sigma}{F}} = \sqrt{\frac{40\cdot10^{-4}\cdot 118750}{300/2}} = 1{,}729$$

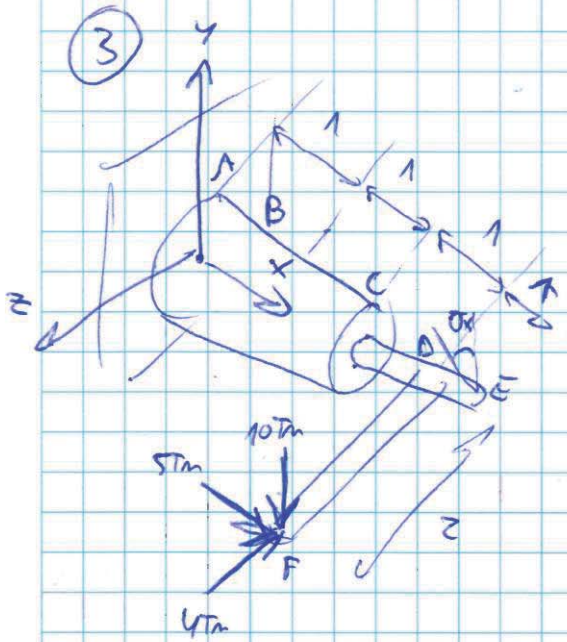
Taula $1{,}729 \Rightarrow \simeq \boxed{\varnothing 29 : \chi}$

$$F\cdot\gamma \leq \varnothing 29 \cdot 40\cdot10^{-4}\cdot\sigma$$

$$\frac{300\cdot 1{,}11}{0{,}29\cdot 40\cdot10^{-4}} = \boxed{284{,}48\ KN}$$ Serà la màxima, per tant haurà de ser més petita

(2)

(3)



Tram AC: $\phi = 40\,cm$

CE: $\phi = 10\,cm$

DE: $\phi = 10\,cm$

$G = 814 \cdot 10^5\ kg/m^2$

Punt A

1.25

Normal (σ_N)

$\Sigma F_x = 0 \quad F_x = 5000 \cdot 9'81 = 49.050\,N$

$\Sigma F_y = 0 \quad F_y = 10.000 \cdot 9'81 = 98.100\,N$

$\Sigma F_z = 0 \quad F_z = 4000 \cdot 9'81 = 39.240\,N$

$\Sigma M_x = 0 \quad M_x = (10000 \cdot 9'81 \cdot 2) = 196'2\,kNm\ (\tau_t)$

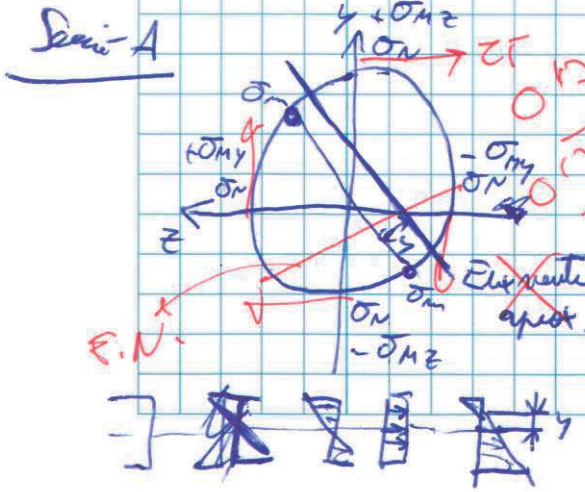
Tallant (τ_V)

$\Sigma M_y = 0 \quad M_y = (5000 + 4000) \cdot 9'81$

$M_y = 5000 \cdot 9'81 \cdot 2 + 4000 \cdot 9'81 \cdot 3 = 215'82\,kNm\ (\sigma_{My})$

$\Sigma M_z = 0 \quad M_z = 10000 \cdot 9'81 \cdot 3 = 294'3\,kNm\ (\sigma_{Mz})$

Secció A



0'47

0'47

$\sigma_N = \dfrac{49050}{\frac{\pi}{4} \cdot 0'4^2} = 0'39\,MPa$ (tracció)

Node (20cm)

$\sigma_{My} = \dfrac{215'82}{\frac{\pi}{64}(0'4^4)} R = 171'74\,MPa \cdot R$

$\sigma_{Mz} = \dfrac{294'300}{\frac{\pi}{64}(0'4^4)} R = 234'19\,MPa \cdot R$

$$\sigma_M = \sqrt{21582^2 + 29430^2}$$

$$\sigma_M = \frac{\sqrt{(21582 \cdot 0'2)^2 + (29430 \cdot 0'2)^2}\ 0'2}{\frac{\pi}{64} \cdot 0'4^4} =$$

$$\sigma_M = \frac{\sqrt{(21582)^2 + 29430^2}\ \cdot 0'2}{\frac{\pi}{64} \cdot 0'4^4} = 58'1 \text{ MPa}$$

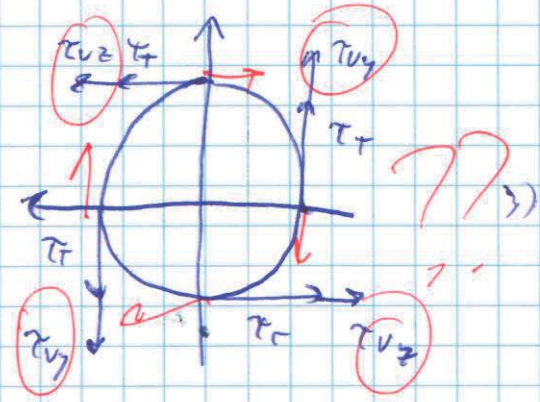
$$\sigma_M = \sigma_M + \sigma_N = 58'1 + 0'39 = \boxed{58'49 \text{ MPa}}$$

$$\tau_t = \frac{M_x \cdot r}{\frac{\pi}{32} d^4} = \frac{19620 \cdot 0'2}{\frac{\pi}{32} \cdot 0'4^4} = \boxed{3'12 \text{ MPa}}$$ del moment

$$\tau_{V_y} = \frac{4}{3} \cdot \frac{F_y}{\pi \cdot 0'2^2} = \frac{4 \cdot 98100}{3 \cdot \pi \cdot 0'2^2} = 1'04 \text{ MPa}$$

$$\tau_{V_x} = \frac{4}{3} \frac{F_z}{A} = \frac{4 \cdot 39240}{3 \cdot \pi \cdot 0'2^2} = 0'416 \text{ MPa}$$

Factor seguretat = 1'456



a)

$$\tau_{max} = 3'12 + 1'456 = \boxed{4'58 \text{ MPa}}$$

b)

$\sigma_N = 0'39$ MPa

$\sigma_{M_y} = 34'348$ MPa

$\sigma_{M_z} = 46'838$ MPa

$\tau_t = 3'12$ MPa

$\tau_{V_y} = 1'04$ MPa

$\tau_{V_x} = 0'416$ MPa

(Calculat abans)

4)

$$\phi = \frac{T \cdot L}{G \cdot I_0} \rightarrow \phi = \frac{4'58 \cdot 10^6 \cdot}{}$$

(Com que son dues barres soldades, rotaran el mateix angle, per tant podem agafar-ne tota la longitud.)

Tallant per l'eix petit:

$$\tau = \frac{196200 \cdot 011}{\frac{\pi}{32} \cdot 011^4} + \frac{4 \cdot 98100}{3 \cdot \pi \cdot 005^2} + \frac{4 \cdot 39240}{3 \pi \cdot 005^2} = 2021'8 \text{ MPa}$$

$$\phi = \phi_{\text{eixAC}} + \phi_{\text{eixCC}} = \frac{4'58 \cdot 10^6 \cdot 2}{012 \cdot 8'24 \cdot 10^{10}} + \frac{2021'8 \cdot 10^6 \cdot 2}{0'05 \cdot 8'24 \cdot 10^{10}} = 0'98 \text{ rad}$$

$$G = \frac{84 \cdot 10^5 \text{ kg}}{\text{cm}^2} \cdot \frac{10000 \text{ cm}^2}{1 \text{ m}^2} \cdot 9'81 = 8'24 \cdot 10^{10} \text{ N/m}^2$$